**Name:** _____ **Date:** _____

# *Math 10F&PC Chapter 4 Roots and Powers*

## *4.5 – Negative Exponents and Reciprocals*

***Focus: relate negative exponents to reciprocals.***

Let's look at the following pairs of numbers:

$5 \times \frac{1}{5} = 1$ $\quad 11 \times \frac{1}{11} = 1$ $\quad \frac{2}{3} \times \frac{3}{2} = 1$

$5, \frac{1}{5}$ $\quad 11, \frac{1}{11}$ $\quad \frac{2}{3}, \frac{3}{2}$ $\quad \frac{1}{15}, 15$

What are so special about each pair of numbers? Each number is the Reciprocal of the other.

***What is Reciprocals?*** when two numbers whose product is 1

***Activity:*** *Complete the following table:*

Ex $\frac{3}{7} \times \frac{7}{3} = 1$

| Question | Expanded Form | Simplified Fraction | Use Quotient Rule | Simplified Exponent |
|---|---|---|---|---|
| $\frac{2^2}{2^3}$ | $\frac{2 \times 2}{2 \times 2 \times 2}$ | $\frac{1}{2}$ | $2^2 \div 2^3 = 2^{2-3}$ | $2^{-1}$ |
| $\frac{3^3}{3^5}$ | $\frac{3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3}$ | $\frac{1}{3 \times 3} = \frac{1}{9}$ | $3^3 \div 3^5 = 3^{3-5}$ | $3^{-2}$ |
| $\frac{0.5^2}{0.5^4}$ | $\frac{0.5 \times 0.5}{0.5 \times 0.5 \times 0.5 \times 0.5}$ | $\frac{1}{0.5^2}$ | 1 | 0.5 |
| $\frac{7^4}{7^9}$ | | | $7^{4-9} =$ | $7^{-5}$ |

What conclusion can you come to looking at the two columns labeled "Simplified Fraction" and "Simplified Exponent"?

$x^{-a} = \frac{1}{x^a}$ Ex: $3^{-2} = \frac{1}{3^2}$ and $\frac{1}{x^{-a}} = x^a$ Ex:- $\frac{1}{7^{-2}} = 7^2$

A negative exponent in numerator becomes a positive exponent in the denominator.

A negative exponent in denominator becomes a positive exponent in the numerator.

$1 \div \frac{1}{2} = 2$

$1 \div \frac{1}{3} = 3$

***Example 1:*** Evaluate each of the following.

a) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

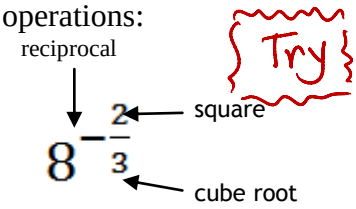
b) $\left(-\frac{3}{4}\right)^{-3} = \frac{1}{\left(-\frac{3}{4}\right)^3} = 1 \times \left(-\frac{4}{3}\right)^3 = \frac{-64}{27}$

c) $0.3^{-4} = \frac{1}{0.3^4} = \frac{1}{0.0081}$

d) $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

Try: $\left(-\frac{4}{3}\right)^{-2} = \frac{1}{\left(-\frac{4}{3}\right)^2} = 1 \times \left(\frac{-3}{4}\right)^2 = \frac{9}{16}$

We can apply both rational and negative exponents together. For example, the rational exponent $8^{-\frac{2}{3}}$ means the following operations:




Try

Ex:- $\left(-\frac{3}{5}\right)^{-3} = \left(-\frac{5}{3}\right)^{3} = \frac{-125}{27}$

In other words:

$8^{-\frac{2}{3}} = 8^{-1\times\frac{1}{3}\times 2}$

Because the order of multiplication doesn't matter, these can be done in any order.

***Example 2:*** Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{8^2}}$ OR $= \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{16}{9}}\right)^3 = \left(\frac{4}{3}\right)^3 = \frac{64}{27}$

c) $16^{-\frac{5}{4}} =$ $\frac{1}{16^{\frac{5}{4}}} = \frac{1}{(\sqrt[4]{16})^5} = \frac{1}{2^5}$

d) $\left(\frac{25}{36}\right)^{-\frac{1}{2}} = \left(\frac{36}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$

e) $\left(\frac{625}{256}\right)^{-\frac{3}{4}} = \left(\frac{256}{625}\right)^{\frac{3}{4}} = \left(\sqrt[4]{\frac{256}{625}}\right)^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$

f) $81^{-0.75} = 81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{\left(\sqrt[4]{81}\right)^3} = \frac{1}{3^3} = \frac{1}{27}$

1. Evaluate $(0.64)^{-\frac{1}{2}}$ without using a calculator.

   a. $\frac{4}{5}$ b. $-\frac{4}{5}$ c. $\frac{1}{4}$ d. $\frac{5}{4}$

   $(0.64)^{-\frac{1}{2}} = \frac{1}{(0.64)^{\frac{1}{2}}} = \frac{1}{\sqrt{0.64}} \times \frac{10}{10} = \frac{10}{8} = \frac{5}{4}$

2. Which power with a negative exponent is equivalent to $\frac{1}{125}$? $= \frac{1}{5^3} = 5^{-3}$

   a. $5^{-3}$ b. $-5^{-3}$ c. $3^{-5}$ d. $(-5)^3$

3. How many times as great as $7^{-2}$ is $7^6$?

   a. $7^{12}$ b. $7^{-8}$ c. $7^4$ d. $7^8$

***Example 3:*** Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, $v$ is the speed in meters per second, $s$ is the distance between successive footprints of the same foot, and $f$ is the foot length in meters. Use the measurements s=1.5m and f=0.3m to estimate the speed of the dinosaur.

$v = 0.155\ s^{\frac{5}{3}}\ f^{-\frac{7}{6}}$ $v = 0.155\ (1.5)^{\frac{5}{3}}\ (0.3)^{-\frac{7}{6}}$

speed = $v$ $v = 1.24$ m/s

***Assignment: p. 233 #7–10, 12, 13, 16***

Home work Quiz